

Discussion

Discussion of paper by J. Krahn and N. R. Morgenstern "The ultimate frictional resistance of rock discontinuities". *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.* 16, 127-133 (1979).

The authors [1] have observed that "two rock samples of the same mineral composition and tested under the same stress state will not have the same large strain shearing resistance if the structure or roughness along the shearing surfaces is not similar". They therefore suggest the use of the term *ultimate* frictional resistance in place of the word *residual*. This terminology was in fact also used some years ago by Krsmanovic [2], who recognised the limits imposed by direct shear testing. He also found that rough bedding planes in limestone were a long way from their residual strength even after several centimeters of shearing.

In the author's tests [1] samples of natural discontinuities of 15 cm length ($A = 225 \text{ cm}^2$) were sheared about 2 cm (13% of their length) before reversing. According to a typical shear load-deformation record, shear resistance gradually reduced to the end of these 4 cm of shearing. All their samples of natural discontinuities were in limestone from Turtle Mountain. Yet because of different roughness their *bedding planes* had ultimate strength of 32° , while their smooth *joints* and *flexural slip surfaces* were down to 14° and 15° respectively.

There may in fact be additional reasons than *initial roughness* for these differences. The state of weathering or joint wall hardness was not described by the authors [1]. Different values of JCS (*joint wall compression strength*) might possibly have been discovered due to the different geological histories of the various discontinuities. Barton & Choubey [3] have in fact found that the *residual friction angle* (ϕ_r) of a joint (the theoretical minimum, with all roughness worn away) is a function of the relative strengths of the joint wall material and the stronger unweathered material in the interior of each block.

$$\phi_r = (\phi_b - 20^\circ) + 20(r/R) \quad (1)$$

where

- ϕ_b = basic friction angle estimated from *tilt* (self weight) tests on dry unweathered sawn surfaces of the particular rock
- R = Schmidt hammer rebound on the dry sawn surfaces (unweathered)
- r = Schmidt hammer rebound on the wet joint surfaces (weathered)

For example a typical rock with $\phi_b = 30^\circ$ and limited weathering (i.e. $r/R = 30/40$) would have a theoretical minimum strength (ϕ_r) of 25° . If the joint weathering had been quite marked (i.e. $r/R = 20/40$), ϕ_r would have been 20° .

The fact that this empirical method was found to give exceptionally accurate predictions of (ϕ_r) does not directly help in the important question of determining how much displacement is required to reach true *residual*, or how far one has progressed towards residual with the usual limited displacement in shear box tests. The important influence of *initial roughness* that was clearly observed by the authors must somehow be quantified.

The proposed method of quantifying initial (specifically peak strength) roughness and its gradual reduction during the course of shearing is based on observations of the shear behaviour of 100 mm long rough tension fractures generated in a weak brittle model material [4]. Figure 1 shows the shear force-displacement behaviour. In these model studies the stress scale was 1:400, and geometric scale 1:300. Both model (m) and

prototype (p) displacements and normal stresses are shown in figure.

It is clear from the continuing dilation seen in the lower half of the figure that at the end of the tests roughness is still contributing to the shear strength. The authors would correctly refer to this as the *ultimate* shear strength.

The *peak strength* (τ) is described by the following equation ref. [3]:

$$\tau = \sigma_n \tan [\text{JRC} \log_{10} (\text{JCS}/\sigma_n) + \phi_r] \quad (2)$$

where

- σ_n = effective normal stress
- JRC = joint roughness coefficient (at peak)
- JCS = joint wall compression strength (measured with a Schmidt hammer ref. [3]).

For the case of these rough model tension joints JRC = 20.0 and JCS = 0.4 MPa. The latter is the same as the unconfined compression strength of the model material since there is no weathering.

Equation 2 can be rearranged so that the roughness mobilized at any displacement can be back-calculated:

$$\text{JRC (mobilized)} = \frac{\arctan(\tau_m/\sigma_n)^\circ - \phi_r^\circ}{\log_{10}(\text{JCS}/\sigma_n)} \quad (3)$$

where τ_m = shear strength mobilized at any displacement.

Equation 3 was evaluated at several points along each of the shear force-displacement curves shown in Fig. 1. Data was then normalized to the form JRC(mobilized)/JRC(peak) and $\delta_n/\delta_n(\text{peak})$, where $\delta_n(\text{peak})$ was the displacement required to reach peak strength under the particular test. This dimensionless data is shown in Fig. 2.

It can be shown that:

$$\frac{\text{JRC(mobilized)}}{\text{JRC(peak)}} = \frac{\arctan(\tau_m/\sigma_n)^\circ - \phi_r^\circ}{\phi_p^\circ - \phi_r^\circ} \quad (4)$$

where $\phi_p = \arctan(\tau_{\text{peak}}/\sigma_n)$.

When JRC(mob.)/JRC(peak) = 0.5, the shear strength mobilized is equal to $\frac{1}{2}(\phi_p + \phi_r)$. In other words shear strength is midway between peak and residual. This point seems to occur at approximately $10 \delta_n(\text{peak})$ for the case of the rough model tension joints. (Smoother joints, or those under the influence of high normal stress, may apparently reach this point at smaller displacements).

The origin of Fig. 2 is given by the simple expression $-(\phi_r/i)$ where $i = \text{JRC(peak)} \cdot \log_{10} (\text{JCS}/\sigma_n)$. Thus for the model tests, with JCS/ σ_n varying from 8.2 to 234 and $\phi_r = 30^\circ$, the origin for the 7 sets of data ranges from -0.6 to -1.6 (approx.).

We are now in a position to approximate the behaviour of the model joints shown in Fig. 2 by a simple table of values, an ideal form for computer simulation. See Table 1.

Numerous tests on rock joints [3] have indicated that $\delta_n(\text{peak})$ is reached *on average* after a shear displacement of about 1% of the length of joint tested (or possibly 1% of the block size for the case of rock masses). The hypothesis that JRC(mobilized) = 0 when shear displacement approaches the length of the specimen ($100 \delta_n(\text{peak})$) is a convenient rule-of-thumb for rough model tension joints. This has not been verified, but it is *apparent* from Fig. 2 that shear strength reduced very slowly beyond τ_{ult} . Shear strength envelopes presented by

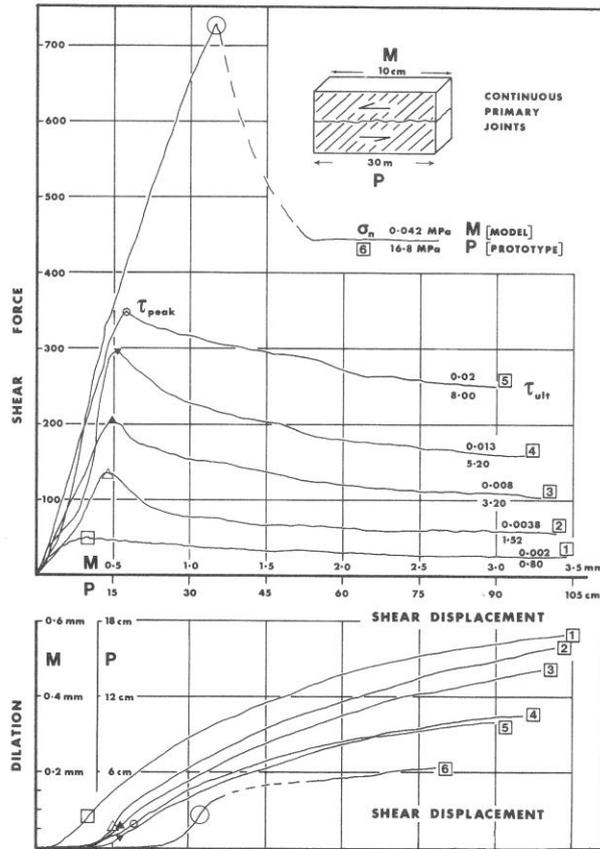


Fig. 1. Shear behaviour of model tension joints.

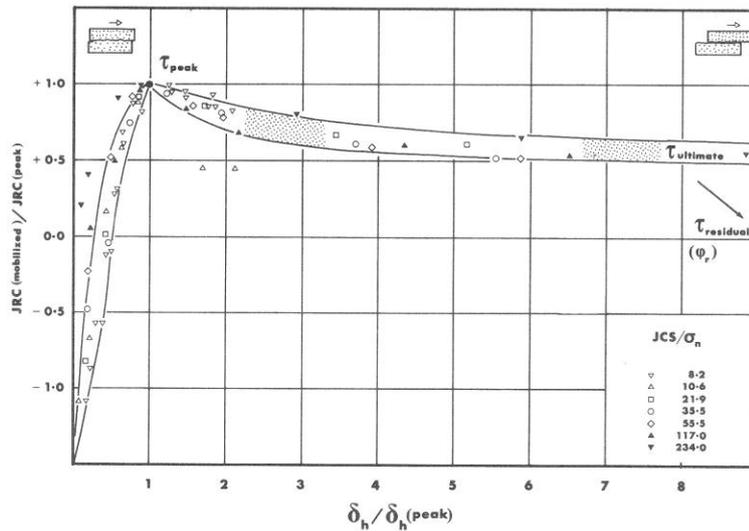


Fig. 2. Model joint behaviour in dimensionless form.

TABLE 1. MOBILIZATION AND REDUCTION OF THE ROUGHNESS WITH SHEAR DISPLACEMENT, FOR THE CASE OF ROUGH MODEL JOINTS

$\delta_h/\delta_h(\text{peak})$	JRC(mobilized)/JRC(peak)
0	$-\phi_r i$
0.5	0.5
1.0 ($=\phi_{\text{peak}}$)	1.0
2.5	0.75
10 ($=\phi_{\text{ult.}}$)	0.5
100 ($\approx\phi_{\text{resid.}}$)	≈ 0

Krsmanovic [2] also suggest that the above may be a reasonable rule-of-thumb for the case of very rough rock joints.

Two examples will now be given that illustrate, like the authors [1], that high initial roughness will result in high ultimate shear strength, all other factors being assumed equal for the present.

Hypothetical examples

A (bedding plane)	B (joint)
$\phi_r = 16^\circ$ (equation 1)	$\phi_r = 16^\circ$
JCS = 100 MPa (Schmidt hammer)	JCS = 100 MPa
JRC = 20 (very rough)	JRC = 10 (medium rough)
$\sigma_n = 2.5$ MPa (test value)	$\sigma_n = 2.5$ MPa
$-\phi_r/i = -0.5$	$-\phi_r/i = -1.0$

Figure 3 illustrates the predicted shear behaviour of these two hypothetical tests. If for the present we define ϕ_{ultimate} as the value of $\arctan(\tau/\sigma_n)$ at a displacement of $10 \delta_h(\text{peak})$, then differences of 8° are indicated, 32° for the bedding plane (example A) and 24° for the joint (example B). According to the previously mentioned rule-of-thumb concerning $\delta_h(\text{peak})$ ($=1\%$ of the joint length) these ultimate strengths would be reached after 1.5 cm of shear displacement for the case of the authors' 15 cm long specimens. The roughness will of course influence this over-simplified '1% rule'.

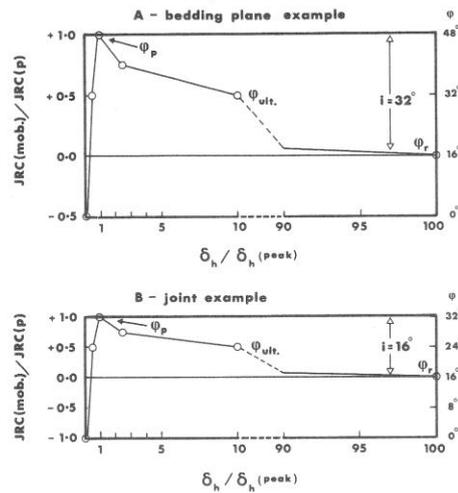


Fig. 3. Demonstration of the effect of initial roughness on ϕ_{ultimate} , the usual limit in shear box tests.

Values predicted for the case of the bedding plane example are quite similar to those found by the authors [1]. However, the value of ϕ_{ultimate} (24°) is too high compared to their results for the joint specimens, where ϕ_{ultimate} was about 15° . This leads the writer to suspect that the values of JCS and ϕ_r were not in fact the same for the joint and bedding plane samples tested by the authors. It is very unlikely that the bedding samples had JRC as high as 20. Reference to Fig. 4 should confirm this. It is then physically impossible that ϕ_r can be as low for the bedding planes as that measured for the joint samples (*ca.* 15°), when peak strength ($\arctan \tau/\sigma_n$) is as high as 60° under the reported levels of normal stress.

A further conclusion one can draw is that, for the same shear displacement, smooth joints are likely to come closer to their true residual (ϕ_r) than rough joints under a given shear displacement. The simplified behaviour given in Table 1 for the case of rough joints may perhaps be improved by acknowledging the multiple influence of JRC, JCS and σ_n on the difficulty or ease with which ϕ_r is approached. Thus in general the value of $i (= \text{JRC} \log_{10} \text{JCS}/\sigma_n)$ will be the determining factor. Small values (i) will induce ϕ_r at smaller displacement than large values of (i). Appropriate values have yet to be determined. As a first approximation it may be expected that ϕ_r is reached at values of $\delta_h/\delta_h(\text{peak})$ in the range 2.5 i -5 i units.

Thus, a fairly smooth joint (JRC = 5) in a weathered rock mass (JCS = 25 MPa) tested under a normal stress of 2.5 MPa will display an (i°) angle of 5° at peak strength. According to the above hypothesis, this 5° of roughness-induced shear strength will be worn away after a shear displacement of $12.5 - 25 \times \delta_h(\text{peak})$. If the sample is assumed to be 100 mm

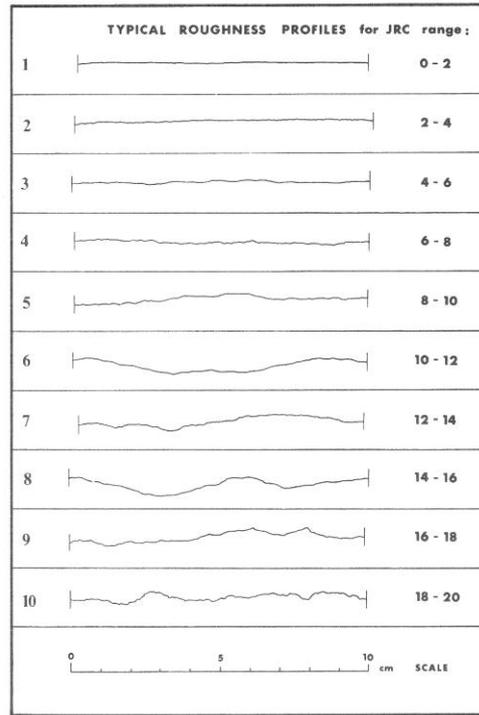


Fig. 4. Profiles of joints with given range of JRC, after Barton & Choubey [3].

long this would mean residual strength reached after a shear displacement in the range 12.5–25 mm.

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REFERENCES

1. Krahn J. & Morgenstern N. R. The ultimate frictional resistance of rock discontinuities. *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.* **16**, 127–133 (1979).
2. Krsmanovic D. Initial and residual shear strength of hard rocks. *Geotechnique* **17**, 145–160 (1967).
3. Barton N. & Choubey V. The shear strength of rock joints in theory and practice. *Rock Mechanics* **10**, 1–54 (1977).
4. Barton N. & Hansteen H. Very large span openings at shallow depth: Deformation magnitudes from jointed models and F.E. analysis. *Proc., Rapid Excavation and Tunneling Conference*, Atlanta (1979).